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Aerodynamic Boundary Layer Effects on Flutter and Damping of Plates

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A previously developed shear flow model for describing the aerodynamic pressures on a moving flexible wall in a boundary layer type flow is utilized to perform stability (flutter) and response analyses for elastic plates. Results for flutter boundaries and damping in the subflutter range are presented and compared to available experimental data. Agreement between theory and experiment is generally good except for damping at low supersonic Mach numbers.

Nomenclature

a, a_∞ = plate length, also speed of sound
b = plate width
D = plate stiffness
E = modulus of elasticity
h = plate thickness
K = $\omega[\rho_m h a^4/D]^{1/2}$
M = U_∞/a_∞, Mach number
q = $\rho_\infty U_\infty{}^2/2$, dynamic pressure
U = air velocity
z = coordinate perpendicular to plate
λ^* = $2qa^3/D$, nondimensional dynamic pressure
μ = $\rho_\infty a/\rho_m h$, air-to-panel mass ratio
δ = boundary layer thickness
ρ = air density
ρ_m = plate density
ω = frequency
ζ = damping ratio

Subscripts

f = flutter
∞ = freestream

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Index categories: Aeroelasticity and Hydroelasticity; Nonsteady Aerodynamics.

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Introduction

IT is now firmly established on experimental grounds that the adjacent fluid shear (boundary) layer can be of importance in modifying the flutter behavior of plates as a result of the recent investigations by Muhlstein, Gaspers and Riddle[1] and Gaspers, Muhlstein and Petroff.[2] The present author has developed a relevant theoretical model which is fully discussed in Ref. 3. Basically the model is an inviscid small perturbation theory with the viscous boundary layer only taken into account through the specification of a nonuniform mean flow. The boundary layer thickness is taken as constant everywhere over the panel. A more complete analysis would include viscous effects in the perturbations as well, i.e., one would employ a dynamic linearization of the Navier-Stokes equations about the same mean flow. Whether this latter model is needed for some applications is still an open question. However the simpler theory of Ref. 3 has already proven useful, and here we explore its potentiality further. Specifically we make comparisons with experimental flutter data over a larger parameter range than that considered in Ref. 3 and also make initial comparisons with experimental data for the aerodynamic damping for conditions below the flutter boundary. By aerodynamic damping we mean the damping provided to the plate by the aerodynamic flow. The structural model is a rectangular isotropic plate clamped on all edges. This paper is a continuation of Ref. 3 where the interested reader may find the mathematical details of the analysis.

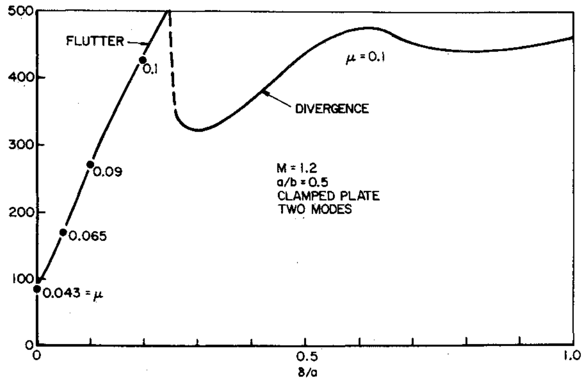

Fig. 1 **Flutter dynamic pressure vs boundary layer thickness.**

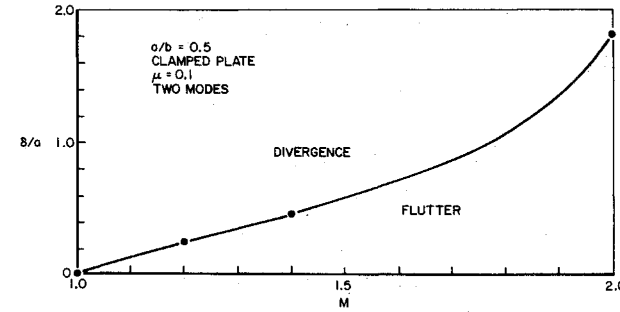

Fig. 3 **Boundary layer thickness vs Mach number.**

Results

Flutter

In Fig. 1 flutter dynamic pressure is given for an extensive range of boundary layer thickness ratio, $\delta/a = 0 \rightarrow 1$. Previously[3] it has been shown that the theoretical results are in good agreement with the available experimental data[1,2] for $\delta/a \leq 0.1$. Note that for $\delta/a \lesssim 0.25$ the flutter boundary abruptly changes; this is associated with a change in the nature of the instability from flutter to static divergence. It would be very valuable to have experimental data for this range of boundary layer thickness. For these data, $a/b = 0.5$, $M = 1.2$ and plate edges are continuously clamped all around.

In Fig. 2 similar theoretical data for $a/b = 2$, $M = 1.2$ and $\delta/a \leq 0.1$ are shown along with the available experimental data. The agreement is satisfactory, and it is seen that the effect of the boundary layer is much less important for $a/b = 2$ than for $a/b = 0.5$.

From such results one can construct plots of boundary layer thickness required to ensure that the panel will statically diverge rather then flutter. In Fig. 3 this boundary layer thickness is plotted vs. Mach number for $a/b = 0.5$ and in Fig. 4 vs a/b for $M = 1.2$. Mach number appears to have the greater influence.

The divergence portion of a stability boundary, such as that shown in Fig. 1, is independent of mass ratio, μ, since it is a static phenomenon. However, the flutter portion of the stability boundary and hence the boundary layer thickness at which a change from flutter to divergence occurs does depend upon μ to some extent. In Fig. 1 which is representative of M, a/b combinations where boundary layer effects are important, the flutter for small δ/a is a single-degree-of-freedom flutter (predominantly in the first mode), as $\delta/a \rightarrow 0.25$ the flutter becomes a coupled mode type (predominantly in the first and second chordwise modes), and for $\delta/a > 0.25$ we have divergence (predominantly in the first mode). The coupled mode flutter is only mildly sensitive to μ although the single-degree-of-freedom flutter is strongly affected, i.e., $\lambda_f \sim \mu$. Hence the intersection point of the flutter-divergence boundaries should not be strongly affected by mass ratio.

Aerodynamic Damping

In solving for the flutter boundary we compute the time history of the panel motion.[3] (If we include the nonlinear structural stiffness, we can also determine the flutter limit cycle amplitude.) Such a procedure also allows us to determine the aerodynamic damping quite simply when for a given combination of parameters flutter does not occur. We prescribe some initial (modal) displacement and observe the (exponential) decay with time, if we are below the flutter boundary. From the time history the damping coefficient, ζ, may be determined using the expression

$$(\exp - \zeta\omega_n t)$$

where ω_n is the nth natural frequency. As the flutter boundary is approached the use of ω_n may be somewhat ambiguous because of significant aerodynamic coupling between modes and consequent modification of the panel natural frequencies by the airstream. This simply means that for coupled modes the concept of a single damping coefficient, ζ, is not as useful. The true damping is still provided by the aerodynamic theory for all dynamic pressure including in the flutter regime itself.

In Fig. 5a some preliminary theoretical results are compared to the available experimental data[4] for a particular boundary layer thickness and $M = 1.4$ and in Fig. 5b theoretical data are given for several δ/a. Several interesting points can be made with these figures.

1) The aerodynamic damping is comparable to or greater than typical values of structural damping, structural $\zeta \approx 0.01$.

2) The effect of a boundary layer of modest thickness, $\delta/a \approx 0.1$, can significantly increase the aerodynamic damping over its inviscid value, $\delta/a = 0$.

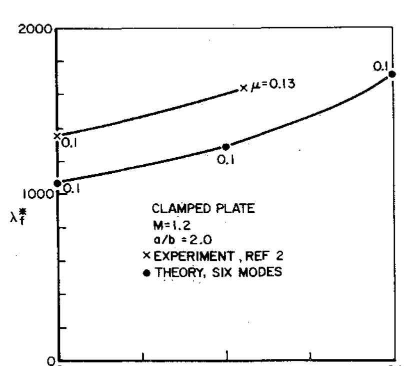

Fig. 2 **Flutter dynamic pressure vs boundary layer thickness.**

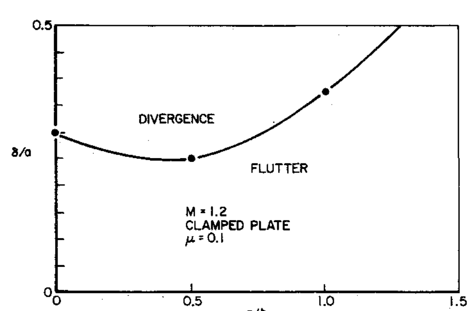

Fig. 4 **Boundary layer thickness vs length to width ratio.**

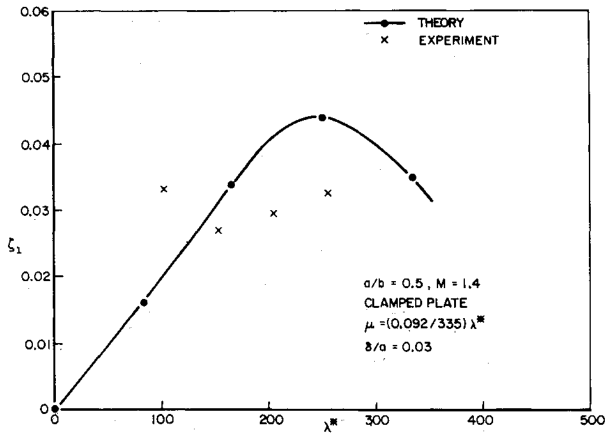

Fig. 5a First mode damping vs dynamic pressure.

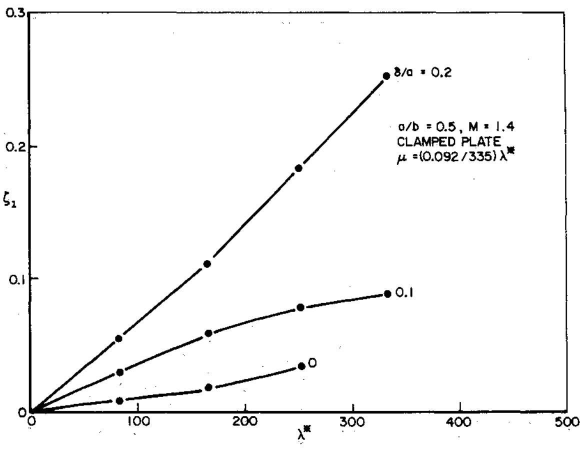

Fig. 5b First mode damping vs dynamic pressure.

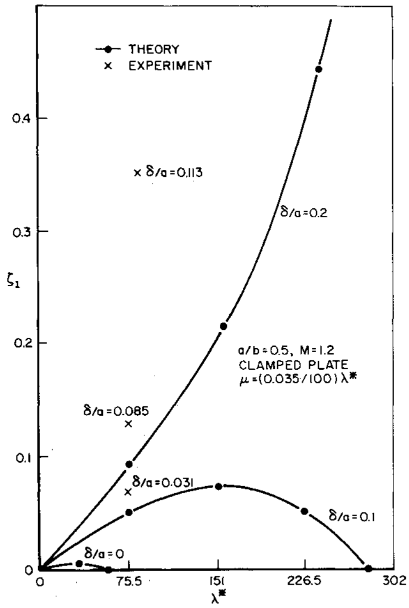

Fig. 5c First mode damping vs dynamic pressure.

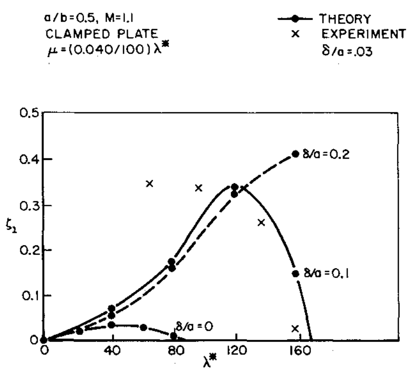

Fig. 5d First mode damping vs dynamic pressure.

3) Theory and experiment are in reasonable agreement for these particular data. It should be emphasized that the experimental data include contributions from structural as well as aerodynamic damping. Hence the experimental data would be expected to be slightly higher than the theoretical data. It should be pointed out that second (chordwise) mode damping is more difficult to define particularly as $\lambda^* \rightarrow \lambda_f^*$ due to aerodynamic coupling with the first mode. Coincidentally for the conditions of Fig. 5a the second and first mode damping are nearly the same for $\lambda^* \lesssim 150$. In general, however, the damping ratios in the two modes differ.

In Figs. 5c and 5d similar results are shown for $M = 1.2$ and 1.1, respectively, including available experimental data. The significance of these data is the clear indication that for lower Mach numbers the agreement between theory and experiment is substantially poorer than for $M = 1.4$. Graphs have been prepared, Figs. 6a and 6b, which give aerodynamic damping as a function of Mach number for various dynamic pressure (and air/panel mass ratio). As may be seen aerodynamic damping varies rapidly with both Mach number and boundary layer thickness as $M \rightarrow 1$. This no doubt, in part, explains the poorer theoretical-experimental correlation under these conditions. Also the results suggest that the effective theoretical boundary layer thickness was nearly $\delta/a = 0.1$ rather than the measured value of 0.031. The measured boundary layer thickness represents an average over the plate length. The actual thickness increases monotonically from plate leading to trailing edge. The maximum thickness (at the trailing edge) is about 20–25% larger than the average for $\delta/a = 0.03$ and less for δ/a greater than 0.03.[5] Whether one can as a general rule simply correct the theory by using a larger effective boundary layer thickness than that actually measured requires further investigation. As can be seen from the above, simply using the trailing edge thickness would still provide too small an effective boundary layer thickness. Of course, the theoretical model not only assumes the boundary layer thickness is constant over the panel length but everywhere for an indefinite distance upstream and downstream. Hence it is possible that there is a long range effect of varying boundary layer thickness off the panel. However Lighthill[6] has investigated the effect of upstream influence for a similar mathematical model in a different physical context and concluded that the distance over which a significant influence exists is on the order of one boundary layer thickness. Hence this does not seem a likely explanation for the discrepancy between theory and experiment either.

Another possible explanation of the discrepancies between theory and experiment is the sensitivity of the results to the mean boundary layer velocity profile assumed for the calculations. In all of the above results a 1/7 power law was used, i.e.,

$$\bar{u}/U_\infty = (z/\delta)^{1/7}$$

and the gradient of the velocity profile obtained by formally differentiating the above

$$d\bar{u}/U_\infty/dz/a = 1/7(z/\delta)^{-6/7}a/\delta$$

There are at least two possible improvements one might consider to this representation. First of all the 1/7 power exponent might be varied. There is some experimental evidence that a 1/9 or even 1/11 exponent might be more accurate. Also the gradient of the mean velocity should drop to zero as $z/\delta \to 1$. This may be accounted for by empirically modifying the expression for $d\bar{u}/U_\infty/dz/a$ to read

$$d\bar{u}/U_\infty/dz/a = 1/7a/\delta[(z/\delta)^{-6/7} - 1]$$

A third improvement one might consider is to modify the above expression so that the gradient of the mean velocity is finite as $z/\delta \to 0$. However the values of z/δ which are encountered in the finite difference solution in z are such (typically $z/\delta > 0.01$) that this refinement does not appear justified.

In Figs. 7a and 7b results are presented analogous to those of Figs. 6a and 6b (for $\delta/a = 0.1$ only) with 1/7, 1/9 and 1/11 power laws. As may be seen while there is an influence of power law exponent, generally it is one of decreasing aerodynamic damping (as the exponent goes to zero we qualitatively expect an approach to the inviscid limit). Hence the use of a 1/9 or 1/11 power law will make the correlation with experiment even poorer although for the dynamic pressure range of the experimental data, $\lambda^* \approx 100$, the effect is not very large.

Next consider the results for modified mean velocity gradients. All results are for the 1/7 power law. Three different mean velocity gradients or slopes are used. The "standard slope" is

$$d\bar{u}/U_\infty/dz/a = 1/7\,\frac{a}{\delta}(z/\delta)^{-6/7}$$

the "smooth slope" is

$$d\bar{u}/U_\infty/dz/a = 1/7a/\delta[(z/\delta)]^{-6/7} - 1]$$

and what we shall call the "zero slope" or "Zeydel ap-

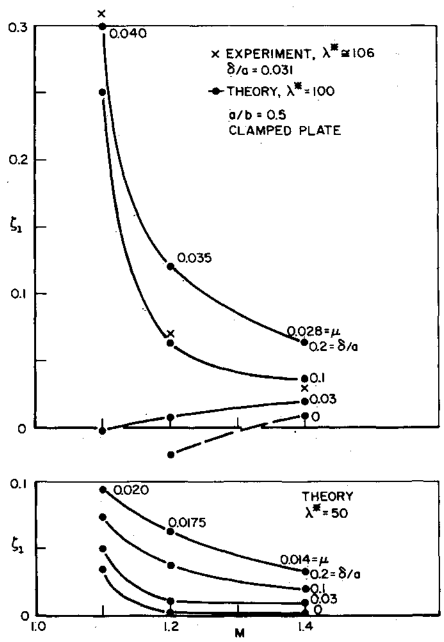

Fig. 6a First mode damping vs Mach number.

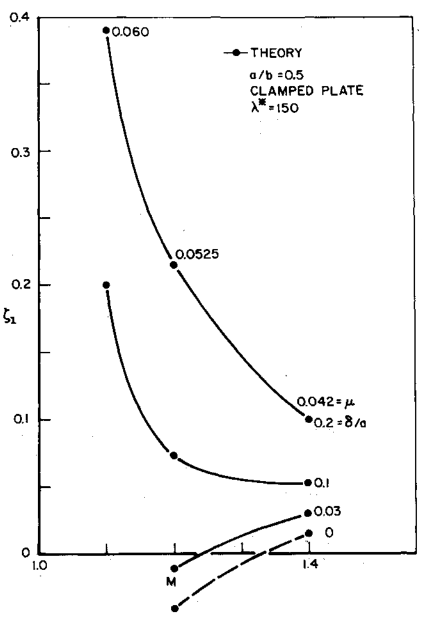

Fig. 6b First mode damping vs Mach number.

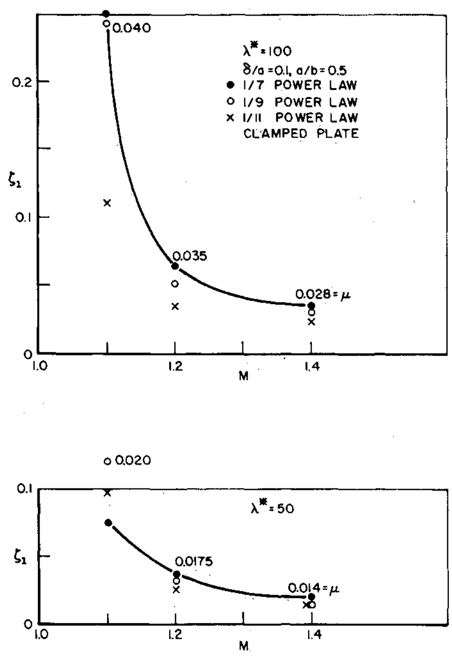

Fig. 7a First mode damping vs Mach number.

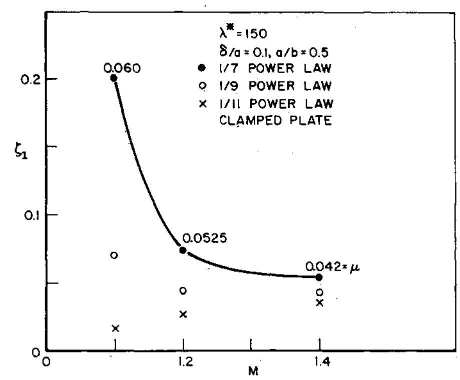

Fig. 7b First mode damping vs Mach number.

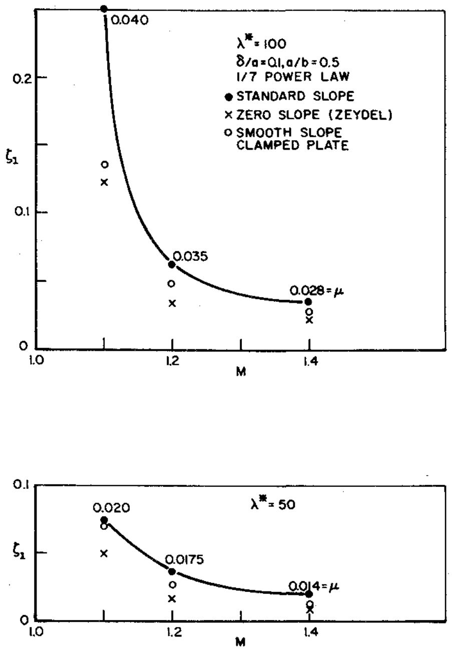

Fig. 8a First mode damping vs Mach number.

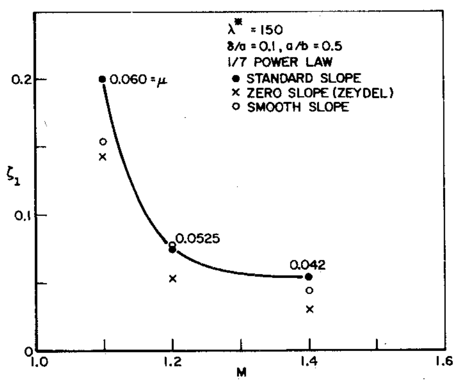

Fig. 8b First mode damping vs Mach number.

proximation" is

$$d\bar{u}/U_\infty/dz/a = 0$$

This is an approximation whereby each different element is treated as a uniform flow without reference to its slope and is similar to the multilayer potential model used by Zeydel[7] based upon an earlier suggestion of Fung.[8]

In Figs. 8a and 8b results are presented analogous to those of Figs. 6a, 6b and 7a, 7b (for $\delta/a = 0.1$ only) with the three different slope approximations. As can be seen the differences between the standard and smooth slope results are modest with the zero slope results differing somewhat more substantially. Presumably the smooth slope results are the most accurate. Again no improvement in theoretical-experimental correlation is indicated as the smooth slope results give smaller damping values than the standard slope calculations, see Fig. 6a.

It is interesting that such relatively small changes are indicated in the damping values due to these various modifications to the mean velocity profile. The changes in the aerodynamic admittance functions (not shown here) may individually undergo much larger changes.

Conclusions

When the aerodynamic boundary layer is included in the analysis (for simple panels) it is shown that several important effects are obtained.

1) For sufficiently large boundary layer thickness the panel diverges rather than flutters. The boundary layer thickness required to obtain divergence increases rapidly with Mach number and less rapidly with length/width ratio. Theoretical-experimental flutter boundaries are in good agreement over the available range of experimental parameters.

2) Below the flutter boundary the aerodynamic flow provides substantial damping to the panel which increases with boundary layer thickness (for supersonic Mach number). Such damping may permit the design of panels of smaller thickness (and less weight) when considering acoustic loads.

3) Theory and experiment are in reasonable agreement at the higher Mach numbers for aerodynamic damping but the agreement becomes poor for lower Mach numbers. The reason for this is still not completely understood, however inclusion of the viscous terms in the small perturbation theory is a logical next step.

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